constituent or constituents presumably belonging to the group of hitherto unrecognized but essential components of an adequate diet.

In the essential features the pathological manifestations described in this investigation closely resemble those which may be observed in human pellagra.

THE COMPLETE ENUMERATION OF TRIAD SYSTEMS IN 15 ELEMENTS

By F. N. Cole, Louise D. Cummings, and H. S. White

Read before the Academy, November 14, 1916

If any set of 15 points are joined by all the possible 105 connecting lines, then these may be combined in sets of three to form 35 triangles. The marks or 'elements' designating the three vertices are a sufficient description of any one triangle, and a list of all 35 triangles constitutes a 'triad system on 15 elements.' Two such sets of triangles are essentially alike if a renaming of its points turns the one list into the other; if that cannot be done, the two lists or systems are essentially different. How many essentially different triad systems can be formed of a given number of elements, is a question of much difficulty, never before answered when the number of elements is 15 or more. For 13 elements there are but 2 different systems, for 9 or 7, only one. The present paper shows that for 15 elements, there are exactly 80 different systems. This conclusive result is established by Mr. Cole.

Three years ago the dissertation of Dr. Cummings increased the number of known triad systems on 15 elements from 10 to 24, and furnished a definite method for comparing systems and verifying their difference or equivalence. All the new systems found by Miss Cummings contained a 'head' —a triad system on 7 of the 15 elements, while one exceptional system among the 10 previously known was 'headless,' a system constructed by Heffter. But all of them admitted groups of transformation into themselves, and it was suspected that the possession of a group might be a necessary property of triad systems. Mr. White takes the group for a starting-point; finds seven types of substitutions, one or more of which must occur in the group; and constructs all the district triad systems for each of those seven typical substitutions. This gives as a gross result 83 systems. By two methods, that of Miss Cummings (by sequences) and one introduced by Mr. White (by trains) these are tested, duplicates are eliminated, and the net result is found to be 44 systems. Of these, exactly 23 exhibit heads and are equivalent to those in Miss Cummings' list, while 21 are headless, including the one such (Heffter's) previously known and twenty new ones. The orders of the groups are 2, 3, 4, 5, 6, 8, 12, 21, 24, 32, 36, 60, 168, 192, 288, and 20160.

Groupless systems on 31 elements were next found (see these PROCEEDINGS, 1, 1915, 4), and the question was raised whether possibly groupless systems could exist in 15 elements. This question is answered by the actual construction of several such systems by Mr. White and Miss Cummings. But while their empirical method is productive, it is not deductive and so could not be shown to be exhaustive. A new starting-point and a new method are requisite, to insure a complete survey of groupless systems as well as the better known kinds. This method was furnished, and its tedious and difficult execution undertaken, by Mr. Cole.

Starting out with the four possible openings:

I.	123	145	167	189	11011	11213	11415
		246	278	2910	21112	21314	2515
II.	123	145	167	189	11011	11213	11415
		246	258	279	21012	21114	21315
III.	123	145	167	189	11011	11213	11415
		246	257	2810	2912	21114	21315
IV.	123	145	167	189	11011	11213	11415
		246	257	2810	2911	21214	21315

which, from the mode of interlacing of the triads containing 1 with those containing 2, may be called dodekad, hexad, single tetrad, and triple tetrad types, respectively, he began by proving that no system could be built up with interlacings of type I alone. Then it was found that with types I and II alone only one system was possible: that already found by Heffter. With this one exception every triad system in 15 letters has an interlacing of type III or IV.

The census was next continued by working out all the systems containing type IV. These included most of the systems with a 7-head (triad system in 7 of the 15 letters); a brief excursus covered the remainder of the 7-head systems. There were in all 23 systems with 7-head and 38 without 7-head.

There remained the systems with type III but not type IV. These were divided into two classes: (1) those with a 'semi-head:' 123 145 167; 246 257; 347; and (2) those without semi-head. Of the former there were 3, of the latter 15.

The total number of types of triad systems in 15 letters therefore provès to be 80.

Proof that the 44 systems with groups are different is based on the set of trains belonging invariantively to each system. There are over

200 kinds of trains altogether, represented in their connection by graphs, so that a glance furnishes intuitive evidence of their essential difference. For the groupless systems Miss Cummings gives a table of the indices which represent the sequences of each system, and comparison is not difficult. She exhibits also a table of the four varieties of interlacing of pairs, as distinguished by Mr. Cole, showing how many of each kind are found in each system. These latter data alone are found, in eight cases, to fail to discriminate two systems actually different. Perfect discrimination would almost certainly be possible by the use of a double entry table, 15 by 15, showing the exact distribution of tetrads, hexads, oktads, and dodekads.

NEW DATA ON THE PHOSPHORESCENCE OF CERTAIN SULPHIDES

(DISCUSSING MEASUREMENTS BY DRS. H. E. HOWE, H. L. HOWES AND PERCY HODGE)

By Edward L. Nichols

DEPARTMENT OF PHYSICS, CORNELL UNIVERSITY[1]

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Ph. Lenard to whom we owe extended studies of the class of highly phosphorescent substances known as the Lenard and Klatt[2] sulphides, describes[3] the spectrum of the emitted light as consisting of a single broad band in the visible spectrum. This band which appears single in most cases, as viewed with the spectroscope, does not however conform to the recognized criteria. The marked difference between the color of fluorescence and that of phosphorescence and the changes of color during decay, suggest over-lapping bands. As shown by E. Becquerel[4] the color of the emitted light varies with the wave length of the exciting rays. His observations apply, it is true to sulphides of barium, calcium and strontium not identical with the preparations of Lenard and Klatt, but belonging to the same class. In a recent paper[5] I gave more direct evidence of the existence of more than one band in the spectra of these substances.

In their original paper[2] Lenard and Klatt depicted these spectra as complex instead of single; but in both the earlier and the later papers attention is given rather to the mode of excitation than to the character of the phosphorescent light itself and the regions of excitation in the violet and ultra violet are carefully mapped.

Significance of the Bands of Excitation.—It seemed probable that these regions of maximum excitation, the positions and appearance of which